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Ignition Analysis of Unpremixed Reactants with Chain Mechanism in a Supersonic Mixing Layer

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Asymptotic analysis is performed to investigate the ignition of a viscous, two-dimensional and supersonic mixing layer of two parallel streams of oxidant and fuel. A three-step schematic kinetic model proposed by Birkan and Law is adopted to grasp the essential properties of full chemistry. Ignition is shown to be characterized by both thermal runaway and chain branching explosion. The lower branch of the characteristic S-shaped curve, corresponding to a nearly frozen regime, is produced by using the critical Damköhler number of dimensionless ignition distance, which consists of Damköhler numbers of chain branching reactions and that of chain termination reaction. The present results show that, in addition to the initial temperature difference of the two streams, the shear parameters and chemical kinetics have strong effects on ignition distance. Even when the velocity difference of two streams is not large, dissipation plays a dominant role as a heat source for ignition. With Mach number increase, ignition moves downstream at first, then reaches a turning point, and finally moves upstream. This analysis also shows that ignition distance will be greatly shortened with an increase of chain-branching reaction rates and will be delayed with an increase of chain termination rate.

Nomenclature

$\bar{a}$ = acoustic velocity
$\bar{B}$ = frequency factor
$\bar{C}_p$ = specific heat at constant pressure
c = $\rho^2 D$
D = diffusion coefficient
D_a = Damköhler number
$\bar{E}$ = activation energy
f = stream function
g = $4\pi\exp(2\eta^2)$
h, I = variable defined in Eq. (7c)
$l, \bar{l}_{10}$ = characteristic length and chemical characteristic length
M = Mach number, $\bar{u}_{10}/\bar{a}_{10}$
M_c = convective Mach number
M_u = $M(1 - u_{20})$
$\bar{Q}$ = chemical heat release
$\bar{R}$ = universal gas constant
T = temperature normalized by $\bar{T}_{\mathrm{ref}}$ in Eq. (3)
T_a = nondimensional activation temperature
u, v = nondimensional velocities in x and y directions, $= \bar{u}/\bar{u}_{\mathrm{ref}}$, $\bar{v}/\bar{u}_{\mathrm{ref}}$, respectively
u_{20} = $\bar{u}_{20}/\bar{u}_{10}$
$\bar{W}$ = molecular weight
w, z = variables defined in Eq. (5)
X = stretched spatial coordinate
x, y = nondimensional space coordinates normalized by $\bar{l}_{\mathrm{ref}}$
Y = mass fraction
α = nondimensional mass fraction of fuel
β = temperature difference, $= T_{10} - T_{20}$
γ = specific heat ratio
δ, Δ = reduced Damköhler number in Eqs. (15) and (16)
ϵ = T_{10}^2/T_a
ζ = variable defined in Eq. (9)
η, ξ = $z/2\sqrt{\xi c}$ and x/l defined in Eq. (5)
θ = temperature in reaction zone
μ = viscosity, $= \bar{\mu}/\bar{\rho}_{10}\bar{u}_{10}l_{10}$
ν = initial mass fraction of radical
ρ = density, $= \bar{\rho}/\bar{\rho}_{10}$
ω = nondimensional chemical reaction rate

Subscripts

F = fuel
ig = ignition
O = oxidizer
R_1, R_2 = radical
ref = reference parameter in Eq. (3)
10, 20 = value of the oxidizer and fuel streamside, respectively

Superscript

$-$ = physical value

Introduction

THE recent interest in scramjet engines for hypersonic vehicles has created a need for efficient analytical methods to predict ignition and combustion in supersonic flows. One of the fundamental models, in which both the detailed chemical kinetics and the complicated transport processes are involved, is the ignition and flame problem in a supersonic mixing layer, especially for hydrogen and oxygen. A number of attempts to understand the mechanisms involved have been made. Classically, Marble and Adamson[1] discussed the ignition lag in the laminar mixing layer of two parallel streams of a cold combustible gas and a hot inert gas. However, the solution is not practical enough due to the introduction of expansion in the streamwise coordinate. Liñán and Crespo[2] analyzed the problem by elucidating the ignition, premixed-flame and diffusion-flame regimes. This approach explained well the complex phenomena but is confined to isovelocity and inviscid flow. Jackson and Hussaini[3] extended this analysis by considering small velocity difference and viscosity but restricted it to a small temperature difference. Furthermore, all the analyses mentioned are based on one-step irreversible reaction kinetics. Although this approach has served combustion theory well, particularly in the context of activation energy asymptotics (Williams[4]), the neglect of radicals has precluded the modeling of many important phenomena.

Recently, efforts have been made to understand the essences of practical processes by using the reduced kinetic mechanism that contains the kinetic information of the elementary mechanism. For example, Niioka[5] studied the chain branching ignition process of a cold combustible gas by a hot inert gas with a two-step Zeldovich-Liñán model. Tam and Ludford[6] augmented this two-step model by an additional competing reaction and studied the extinction. Birkan and Law[7] also analyzed the structure and extinction of diffusion flames with another

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three-step chain mechanism, which explained the extinction phenomenon quite well. On the other hand, as an example of refined reduced kinetic models, Peter and Williams[8] and Peter and Kee[9] deduced a three-step chemistry and a four-step chemistry through introduction of steady and partial equilibrium approximations and studied the structure of methane-air flames. Also using the reduced kinetic mechanism, Treviño and Méndez[10] analyzed the boundary-layer ignition of hydrogen/oxygen mixture by a hot flat plate. All of these analyses are restricted to subsonic incompressible flow. Only one paper, written by the present authors[11], is concerned with the ignition analysis of compressible supersonic mixing layer with branched chain reaction. However, only the premixed gas is considered.

The object of the present study is to analyze the ignition of a viscous, two-dimensional, spatially developing, and supersonic mixing layer of two parallel streams of unpremixed oxidant and fuel, undergoing a three-step reaction kinetics proposed by Birkan and Law[7] and also to predict the ignition distance and to discuss the effects of free shear, Mach number, and reaction rates on the ignition distance.

Formulation

The origin of the present problem lies in the recent interest in the development of air-breathing scramjet engines. In flight, Mach numbers up to 20 and up to 8 or more in the combustors are expected. Inlet air temperatures will be 1400-2500 K. Because of the interest in the properties of combustors for high speed flow, a basic examination of ignition lag controlled by mixing processes and chemical kinetics is necessary. The present contribution considers a typical case for high speed mixing layers as shown in Fig. 1. A hot oxidizer stream mixes with a cool fuel stream at $x = 0$, with temperatures $\bar{T}_{10}$ and $\bar{T}_{20}$, mass fractions $\bar{Y}_O$ and $\bar{Y}_F$, and velocities $\bar{u}_{10}$ and $\bar{u}_{20}$, respectively, both semi-infinite and supersonic.

A nonreacting supersonic mixing layer has been explored experimentally by Papamoschou and Roshko,[12] who showed that the growth rate decreases with the increase of convective Mach number M_c because of the reduced amplification of the disturbance. Sekar and Mukunda's[13] calculation shows that, in velocity difference of 830 m/s for $M_c = 0.76$, the Reynolds number based on the velocity difference is about 4×10^2. This value may be low enough to consider the flow to be laminar. In the present study, we assume that the velocity difference is not too large, then it is justifiable to consider the flow to be laminar and shock free.

To understand the essential physical and chemical processes, we use the three-step, irreversible schematic model proposed by Birkan and Law.[7]

$$F + R_1 \rightarrow 2R_2 \tag{1a}$$

$$O + R_2 \rightarrow 2R_1 \tag{1b}$$

$$R_1 + R_2 + M' \rightarrow 2P + M' + \bar{Q} \tag{1c}$$

where F, O, P, and M' designate fuel, oxidizer, product, and an inert third body, respectively; and R_1 and R_2, are the intermediate radicals. The first two reactions (1a) and (1b) are chain-branching reactions with negligible chemical heat release and high-activation energies that require radicals to proceed and, in turn, generate more radicals. The third one (1c) is a exothermic termination with zero activation energy. Treating these three reactions as elementary ones, their reaction rates $\bar{\omega}_i$ can be written as

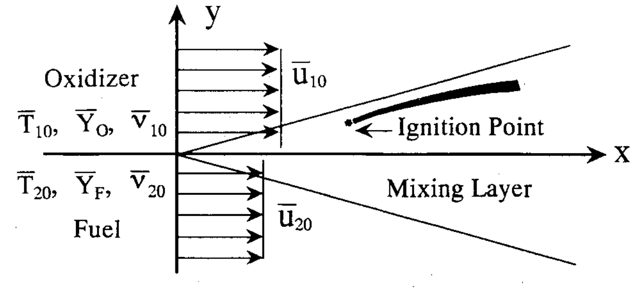


Fig. 1 Schematic illustration of the problem.

$$\bar{\omega}_1 = \bar{B}_1\bar{\rho}^2 \frac{\bar{Y}_F\bar{Y}_{R1}}{\bar{W}_F\bar{W}_{R1}} \bar{T}^2 \exp(-\bar{E}/\bar{R}\bar{T})$$

$$\bar{\omega}_2 = \bar{B}_2\bar{\rho}^2 \frac{\bar{Y}_O\bar{Y}_{R2}}{\bar{W}_O\bar{W}_{R2}} \bar{T}^2 \exp(-\bar{E}/\bar{R}\bar{T})$$

$$\bar{\omega}_3 = \bar{B}_3\bar{\rho}^3 \frac{\bar{Y}_{R1}\bar{Y}_{R2}}{\bar{W}_{M'}\bar{W}_{R1}\bar{W}_{R2}} \bar{T}^3$$

With conventional transport property assumptions (unit Prandtl number and unit Lewis number), the nondimensional governing equations with zero pressure gradient can be written as following:

$$\begin{aligned}
&\rho T = 1 \\
&\nabla \cdot (\rho v) = 0 \\
&L\{u\} = 0 \\
&L\{T\} = M^2(\gamma - 1)T_{10}\mu\left(\frac{\partial u}{\partial y}\right)^2 + \omega_3 \\
&L\{Y_F\} = -\omega_1 \\
&L\{Y_O\} = -\omega_2 \\
&L\{Y_{R1}\} = -\omega_1 + 2\omega_2 - \omega_3 \\
&L\{Y_{R2}\} = 2\omega_1 - \omega_2 - \omega_3
\end{aligned} \tag{2a}$$

Subject to the boundary conditions for $x = 0$, $y > 0$, $x > 0$, $y \rightarrow +\infty$

$$T = T_{10}, \qquad Y_O = 1, \qquad Y_F = Y_{R2} = 0$$

$$Y_{R1} = \nu_{10}, \qquad u = 1, \qquad v = 0$$

for $x = 0$, $y < 0$ and $x > 0$, $y \rightarrow -\infty$

$$\begin{aligned}
&T = T_{20}, \qquad Y_F = \alpha, \qquad Y_O = Y_{R1} = 0 \\
&Y_{R2} = \nu_{20}, \qquad u = u_{20}, \qquad v[vi:] = 0
\end{aligned} \tag{2b}$$

where

$$L\{\cdot\} = \left[\rho v \,.\, \nabla - \frac{\partial}{\partial y}\left(\mu\frac{\partial}{\partial y}\right)\right]\{\cdot\}$$

and the various reference parameters are given by

$$\bar{T}_{\text{ref}} = \frac{\bar{Q}\bar{Y}_O}{\bar{C}_p\bar{W}_O}, \qquad \bar{Y}_{\text{ref}} = \frac{\bar{Y}_O\bar{W}_j}{\bar{W}_O}, \qquad \bar{T}_a = \frac{\bar{E}}{\bar{R}\bar{T}_{\text{ref}}}$$

$$\mu = \frac{\bar{\mu}}{\bar{l}_{10}\bar{\rho}_{10}\bar{u}_{10}}, \qquad \bar{u}_{\text{ref}} = \bar{u}_{10}, \qquad \bar{\rho}_{\text{ref}} = \bar{\rho}_{10}, \qquad \bar{l}_{\text{ref}} = \bar{l}_{10} \tag{3}$$

$$j = F, O, R_1, R_2$$

The nondimensional reaction rates are

$$\begin{aligned}
&\omega_1 = D_{a1}Y_FY_{R1}\exp(-T_a/T) \\
&\omega_2 = D_{a2}Y_OY_{R2}\exp(-T_a/T)
\end{aligned} \tag{4}$$

$$\omega_3 = D_{a3} Y_{R1} Y_{R2}$$

where

$$D_{a1} = \bar{B}_1 \frac{\bar{l}_{10}\bar{\rho}_{10}\bar{Y}_O^3\bar{Q}^2}{\bar{u}_{10}\bar{W}_O^3\bar{C}_p^2}$$

$$D_{a2} = \bar{B}_2 \frac{\bar{l}_{10}\bar{\rho}_{10}\bar{Y}_O^3\bar{Q}^2}{\bar{u}_{10}\bar{W}_O^3\bar{C}_p^2}$$

$$D_{a3} = \bar{B}_3 \frac{\bar{l}_{10}\bar{\rho}_{10}^2\bar{Y}_O^4\bar{Q}^3}{\bar{u}_{10}\bar{W}_O^4\bar{W}_{M'}\bar{C}_p^3}$$

are the Damköhler numbers of reactions (1a), (1b), and (1c), respectively, representing the ratio of characteristic flow time to characteristic time of each reaction. Here, ν_{10} and ν_{20} denote the initial mass fraction of radicals that are needed for the initiation of chain branching reactions. In the present model, the initial values of ν_{10} and ν_{20} are assumed to be two orders smaller than the concentration of reactants, that is, $\mathcal{O}(\epsilon^3)$, to simulate the recirculation zone in scramjet engines. If the initial values are very low, a fourth reaction $F \rightarrow R_2$ is needed because the branching reactions (1a) and (1b) are extremely slow. To obtain the equations in a form compatible with similar solutions, the customary Howarth-Dorodnitsyn transformation is introduced

$$\xi = \frac{x}{l}, \qquad z = \frac{1}{\sqrt{l}} \int_0^y \rho \, \mathrm{d}y, \qquad w = \sqrt{l}\left(\rho v + u \int_0^y \frac{\partial \rho}{\partial x} \mathrm{d}y\right) \tag{5a}$$

The transformed equations may also be written in terms of the variables ξ and η

$$\eta - \eta_0 = z/2\sqrt{\xi c}, \qquad f'(\eta) = u(\eta), \qquad w = \sqrt{c}(\eta f' - f)/\sqrt{\xi} \tag{5b}$$

Thus, the velocity, temperature, and mass fractions can be determined from the equations

$$f''' + 2ff'' = 0 \tag{6a}$$

$$4\xi f' T_\xi - 2fT_\eta - T_{\eta\eta} - T_{10}M^2(\gamma - 1)(f'')^2 = 4\xi l \omega_3/\rho \tag{6b}$$

$$4\xi f' Y_{j_\xi} - 2fY_{j_\eta} - Y_{j_{\eta\eta}} = -4\xi l \omega_j/\rho \tag{6c}$$

$$\begin{aligned} \omega_F &= -\omega_1, \qquad \omega_O = -\omega_2 \\ \omega_{R1} &= -\omega_1 + 2\omega_2 - \omega_3 \\ \omega_{R2} &= -\omega_2 + 2\omega_1 - \omega_3 \end{aligned} \tag{6d}$$

Here we have assumed that $c = \rho^2 D$ is constant to represent the linear viscosity law.

Equations (6) do not have similarity solutions due to the presence of the nonlinear source terms. However, by using the Shvab-Zeldovich approach, coupling functions $T - Y_F + 2Y_O + Y_{R1}$ and $T - Y_O + 2Y_F + Y_R$ can be obtained, yielding

$$\begin{aligned} T - Y_F + 2Y_O + Y_{R1} &= T_{10} + 2 + \nu_{10} \\ &+ (T_{20} - \alpha - T_{10} - 2 - \nu_{10})h \\ &+ (\gamma - 1)M^2 T_{10}(1 - u_{20})^2 h(1 - h)/2 \end{aligned} \tag{7a}$$

$$\begin{aligned} T - Y_O + 2Y_F + Y_{R2} &= T_{10} - 1 \\ &+ (T_{20} + 2\alpha - T_{10} + 1 + \nu_{20})h \\ &+ (\gamma - 1)M^2 T_{10}(1 - u_{20})^2 h(1 - h)/2 \end{aligned} \tag{7b}$$

where

$$h = \frac{I(\infty) - I(\eta)}{I(\infty) - I(-\infty)}$$

$$I(\eta) = \int_0^\eta \exp\left[-\int_0^t 2f(s)\,\mathrm{d}s\right]\mathrm{d}t \tag{7c}$$

Asymptotic Solution

Outer Solution

Our purpose here is to find an asymptotic solution in the limit of large activation energy ($T_a \rightarrow \infty$) with Y_O, Y_F, and T_∞ being of order unity and with Damköhler numbers ranging from zero to a finite value. In the range of $T_a \gg 1$, chemical reactions occur only in a thin region of $\mathcal{O}(\epsilon)$ around the location of maximum temperature. Within the reaction layer, the value of η is very large because of the temperature difference.[2] In the broad outer regime, the branching reactions are effectively suppressed because of their temperature-sensitive Arrhenius kinetics. The exponential term should be very small compared to the convection and diffusion terms.

For large values of η, the stream function can be expanded in

$$f = \eta + \frac{u_{20} - 1}{\sqrt{8\pi}(1 + u_{20})} \frac{e^{-\eta^2}}{\eta^2}\left(1 - \frac{3}{2\eta^2} + \ldots\right) \tag{8}$$

and this shows that it is justifiable to use $f \approx \eta$ as a simplification. By introducing

$$\zeta = [1 - \exp(\eta)]/2, \qquad M_u = M(1 - u_{20}), \qquad \beta = T_{10} - T_{20} \tag{9}$$

and considering that the temperature distributions of the frozen region are perturbed from their zeroth solutions by a small amount of order ϵ, the following expressions are obtained for the first-order perturbations from Eqs. (6)

$$T^- = T_{10} + a_1^-\zeta + a_2^-\epsilon\zeta + (\gamma - 1)M_u^2 T_{10}\zeta(1 - \zeta)/2 + \mathcal{O}(\epsilon^2) \tag{10a}$$

$$\begin{aligned} T^+ &= T_{10} - \beta + a_1^+(1 - \zeta) + a_2^+\epsilon(1 - \zeta) \\ &+ (\gamma - 1)M_u^2 T_{10}\zeta(1 - \zeta)/2 + \mathcal{O}(\epsilon^2) \end{aligned} \tag{10b}$$

$$Y_F^- = b_1^-\zeta + b_2^-\epsilon\zeta + \mathcal{O}(\epsilon^2) \tag{10c}$$

$$Y_F^+ = \alpha + b_1^+(1 - \zeta) + b_2^+\epsilon(1 - \zeta) + \mathcal{O}(\epsilon^2) \tag{10d}$$

$$Y_O^- = 1 + c_1^-\zeta + c_2^-\epsilon\zeta + \mathcal{O}(\epsilon^2) \tag{10e}$$

$$Y_O^+ = c_1^+(1 - \zeta) + c_2^+\epsilon(1 - \zeta) + \mathcal{O}(\epsilon^2) \tag{10f}$$

where the superscript minus denotes the solution of the fuel side and the superscript plus that of the oxidizer side. Since most of the radicals generated in the reaction zone are combined quickly with each other, the radical leakage to the frozen region is very small. Substituting the perturbations into Eqs. (7) and applying the preceding requirement to the profiles of $Y_{R1}^\pm$ and $Y_{R2}^\pm$, we can obtain eight equations to decide $b_1^\pm$, $b_2^\pm$, $c_1^\pm$, and $c_2^\pm$.

Inner Solution

Since the velocity difference is not too large, $f \approx \eta$ is a practical and simple approximation for the reaction zone where η has a fairly large positive value. By using the function ζ and neglecting the first term compared with other terms[12] in this nearly frozen ignition regime, Eqs. (6) can be rewritten as

$$\frac{\mathrm{d}^2 T}{\mathrm{d}\zeta^2} = -g(\zeta)\frac{l}{\rho}\xi D_{a3} Y_{R1} Y_{R2} \tag{11a}$$

$$\frac{d^2 Y_F}{d\zeta^2} = g(\zeta)\frac{l}{\rho}\xi D_{a1} Y_F Y_{R1}\exp(-T_a/T) \tag{11b}$$

$$\frac{d^2 Y_O}{d\zeta^2} = g(\zeta)\frac{l}{\rho}\xi D_{a2} Y_O Y_{R2}\exp(-T_a/T) \tag{11c}$$

where $g(\zeta) = 4\pi\exp(2\eta^2)$. In the reaction region of thickness of $\mathcal{O}(\epsilon)$, temperature rise is of order $\mathcal{O}(\epsilon)$ from the frozen temperature profile. Studies from chemical kinetics have demonstrated that radicals usually exist in low concentration before chain branching explosions. It may be reasonable to assume that the radical concentrations are about one order smaller than the reactant concentration in the reaction zone. When the stretched spatial coordinate $X = \zeta/\epsilon$ is introduced, the temperature and the reactant distributions are expanded as

$$T(X) = T_{10} + \epsilon[\theta(X) - \beta X] + \epsilon(\gamma - 1)M_u^2 T_{10} X/2 + \mathcal{O}(\epsilon^2) \tag{12a}$$

$$Y_O(X) = 1 + \epsilon[y_O(X) - X] + \mathcal{O}(\epsilon^2) \tag{12b}$$

$$Y_F(X) = \epsilon[y_F(X) + \alpha X] + \mathcal{O}(\epsilon^2) \tag{12c}$$

$$(Y_{R1}, Y_{R2}) = \epsilon^2[\Delta_1^{-1} y_{R1}(X), \Delta_2^{-1} y_{R2}(X)] \tag{12d}$$

Substituting these inner expansions into Eqs. (11), we obtain

$$X^2\frac{d^2 y_F}{dX^2} = \xi(y_F + \alpha X) y_{R1}\exp(\theta - \beta_0 X) \tag{13a}$$

$$X^2\frac{d^2 y_O}{dX^2} = \xi[1 + \epsilon(y_O - X)] y_{R2}\exp(\theta - \beta_0 X) \tag{13b}$$

$$X^2\frac{d^2\theta}{dX^2} = -\delta\xi y_{R1} y_{R2} \tag{13c}$$

where

$$\begin{aligned} y_{R1} &= \Delta_1(y_F - 2y_O - \theta)/\epsilon \\ y_{R2} &= \Delta_2(y_O - 2y_F - \theta)/\epsilon \end{aligned} \tag{14}$$

$$\Delta_1 = \frac{g(\zeta_0) D_{a1} l \epsilon^2 \exp(-T_a/T_{10})}{\rho_{10}}$$

$$\begin{aligned} \Delta_2 &= \frac{g(\zeta_0) D_{a2} l \epsilon^2 \exp(-T_a/T_{10})}{\rho_{10}} \\ \Delta_3 &= \frac{g(\zeta_0) D_{a3} l \epsilon^3}{\rho_{10}} \end{aligned} \tag{15}$$

$$\begin{aligned} \delta = \Delta_3/\Delta_1\Delta_2, \qquad & \beta_0 = \beta - (\gamma - 1)M_u^2 T_{10}/2 \\ g(\zeta_0) &= -2/\ell n(4\pi\epsilon^2) \end{aligned} \tag{16}$$

It is clear that Δ_1, Δ_2, and Δ_3 are the relevant reduced Damköhler numbers of reactions (1a–1c), representing the ratios of characteristic flow time to the characteristic reaction time of each of these equations. As Birkan and Law[7] have done, we introduce δ as a measure of the overall efficiency of the branching to the termination reaction and β_0 as a measure of nondimensional temperature difference and dissipation.

In the limit of $\epsilon \to 0$, the numerators of Eqs. (14) should tend to zero in order that the radical concentrations remain finite. Using this requirement and matching the inner solution with the outer solution, we can obtain an equation that is exactly the same form as that describing the nearly frozen ignition regime of Liñán and Crespo[2] and Niioka[14] for a one-step overall reaction, that is,

$$\hat{X}^2\frac{d^2\hat{\theta}}{d\hat{X}^2} = -\hat{\xi}(\hat{X} - \hat{\theta})\exp(\hat{\theta} - \hat{\beta}\hat{X}) \tag{17a}$$

$$\hat{\theta}\bigg|_{\hat{X}=0} \to 0, \qquad \frac{d\hat{\theta}}{d\hat{X}}\bigg|_{\hat{X}\to\infty} \to 0 \tag{17b}$$

where

$$\begin{aligned} \hat{\theta} = 2\theta, \qquad \hat{X} = 2\alpha X, \qquad & \hat{\xi} = \epsilon\xi/\delta \\ \hat{\beta} = [\beta - (\gamma - 1)M_u^2 T_{10}/2]/\alpha & \end{aligned} \tag{18}$$

and the radical concentrations in the reaction layer are

$$\begin{aligned} y_{R1} &= [1 - (\theta + X)]\exp(\theta - \beta_0 X)/\delta \\ y_{R2} &= \epsilon(-\theta + \alpha X)\exp(\theta - \beta_0 X)/\delta \end{aligned} \tag{19}$$

Thus, by using $\hat{\xi}_{ig}$, the dimensional ignition distance can be represented as

$$\bar{x}_{ig} = \bar{l}_{10} M \bar{\xi}_{ig} \tag{20}$$

where

$$\bar{l}_{10} = \frac{\bar{B}_3 \bar{W}_O^2 \bar{C}_p \bar{a}_{10} T_a^2 \rho_{10} \exp(2T_a/T_{10})}{\bar{B}_1 \bar{B}_2 \bar{W}_{M'} \bar{Q} \bar{Y}_O^2 g(\zeta_0) T_{10}^4}$$

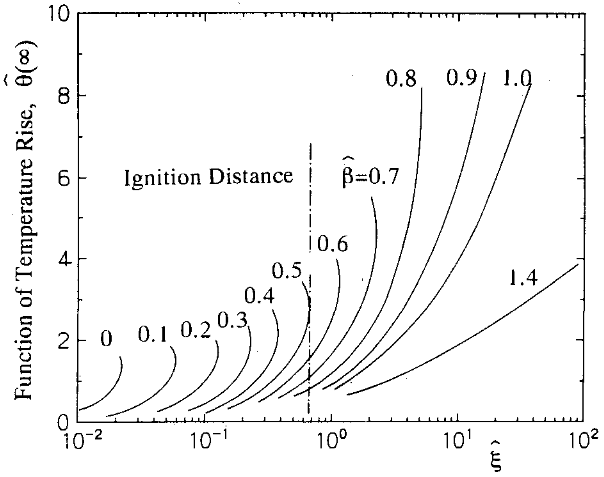
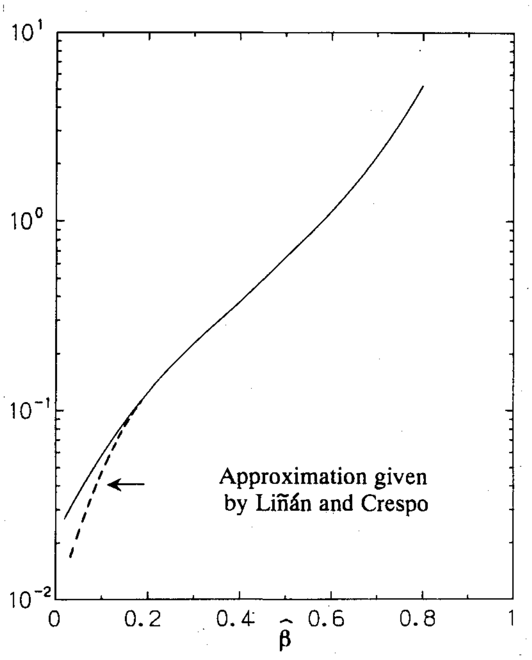


Fig. 2 Maximum perturbed temperature $\hat{\theta}(\infty)$ vs $\hat{\xi}$ for various $\hat{\beta}$.



Ignition Location, $\hat{\xi}_{ig}$, vs $\hat{\beta}$

Fig. 3 Ignition distance $\hat{\xi}_{ig}$ vs $\hat{\beta}$.

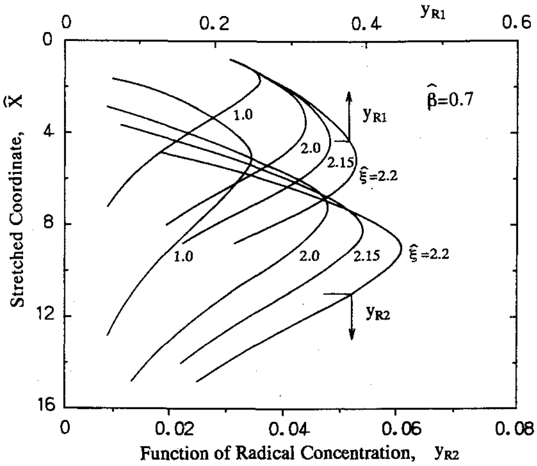


Fig. 4 Concentration profiles of radical R_1 and R_2 for $\hat{\beta} = 0.7$ and various ξ.

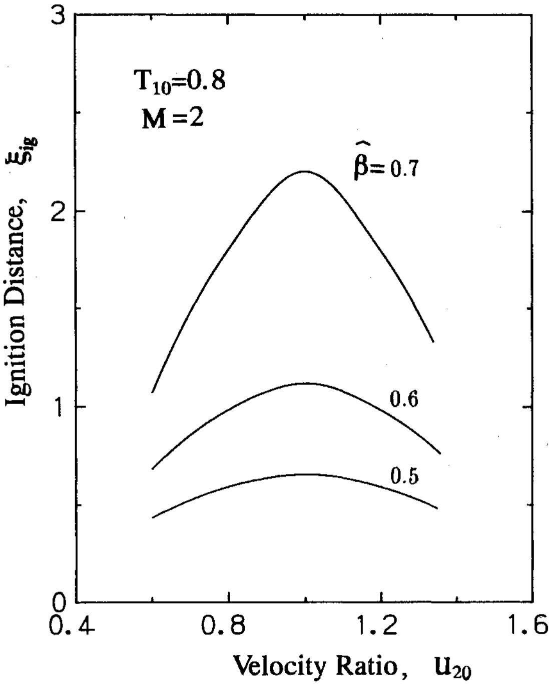


Fig. 5 Effect of velocity difference on ignition distance with difference reaction heat release.

representing the reference length as a measure of the chemical characteristic length.

Results and Discussion

Equations (17) have been solved numerically by using the quasilinearization technique. Figure 2 exhibits the lower bend of the S-response curve produced by plotting the maximum temperature $\hat{\theta}(\infty)$ as a function of ξ for various $\hat{\beta}$, where ξ is the nondimensional ignition distance and $\hat{\beta}$ consists of both the initial temperature difference and dissipation term nondimensionalized by chemical heat release. The response curves of the considered combustion system clearly display ignition phenomenon at the points where the curves turn around. This means that the vertical tangents of these curves at the turning points are the critical ignition distance. It is shown that the temperature rise at the turning point decreases with the increase of the fuel stream temperature.

Figure 3 shows the ignition distance plotted as a function of parameter $\hat{\beta}$. Liñán and Crespo[2] have shown that an excellent approximation for ξ_{ig} is $2e^{-2}\hat{\beta}(2-\hat{\beta})/(1-\hat{\beta})^2$. They pointed out that the character of the ignition stage depends on the difference of the initial temperature of the reactants. In the present analysis, the parameter $\hat{\beta}$ here contains the contribution of viscous heating to the ignition, we fill find that this viscous heating plays an important role in the supersonic mixing layer as well as the initial temperature difference. It should be noted that the present analysis is only valid for $T_{10} - T_{20} = \mathcal{O}(1)$. The case of $T_{10} - T_{20} = \mathcal{O}(\epsilon)$ needs another analysis because the reaction can occur at the whole domain of the mixing layer.

Figure 4 shows the development of the concentration of radicals. It is seen that both radical R_1 and radical R_2 increase quickly along the stream. Reaction is initiated at the high temperature region because of the temperature-sensitive Arrhenius kinetics, even though the concentration of fuel is very low there. However, after the initiation, a higher fuel concentration is needed to accelerate the branching reaction. Therefore, the reaction zone moves gradually to the low temperature side and generates more and more radicals with the development of streams. Around $\xi = 2.2$ in Fig. 4 there exists a turning point for maximum radical concentrations, like temperature, representing the branching explosion.

It is of interest to investigate the effects of shear parameters. In Fig. 5 we plotted the nondimensional ignition distance as a function of velocity difference for various $\hat{\beta}$. With constant initial temperatures, it is seen that ignition is greatly delayed with the decrease of chemical heat release and is accelerated due to the existence of velocity difference. For small reaction heat release, the temperature rise in the reaction zone only depends on the convection and diffusion, which makes the initiation of the branching reaction difficult. On the other hand, a larger velocity difference enhances the mixing and induces temperature rise by dissipation, causing ignition to occur sooner. It is obvious that this effect is much more important when chemical heat release is small. Therefore, as shown in Fig. 5, the ignition distance should be greatest when the velocity difference is zero; and, accordingly, there is no convection normal to the main flow and no dissipation.

Figure 6 shows the dependence of ignition distance $\bar{x}_{ig}/\bar{l}_{10}$ on the Mach number for typical velocity differences. It is seen that when the velocities are almost equal, ignition distance increases almost linearly with Mach number. When the velocity difference is large, the ignition point moves downstream at first, then reaches the turning point, and finally moves upstream with an increase of Mach number. It follows that an increase of Mach number, while keeping constant velocity ratio, leads to an increase of temperature, which feeds more energy into the system causing it to ignite sooner.

It is shown that ignition distance is proportional to $\bar{B}_3/\bar{B}_1\bar{B}_2$. Therefore, both the branching reactions and the termination reaction have strong effects on the ignition delay. Ignition will be accelerated by an increase of branching reaction rates and will be delayed by an increase of termination reac-

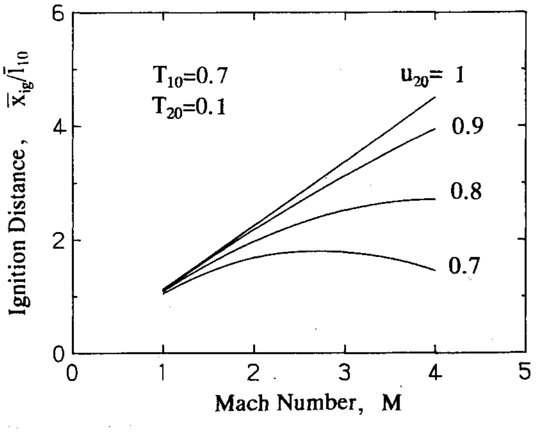


Fig. 6 Effect of Mach number on ignition distance for typical velocity difference.

tion rate since the increase of termination reaction rate will decrease radical concentration and suppress the branching reaction in reverse.

Conclusion

In the present study, the ignition of two reactants in a supersonic mixing layer undergoing a chain reaction mechanism is asymptotically analyzed. Attention is paid to the branched chain chemical kinetics, shear parameters, and temperature difference; this is a measure of departure from the one-step overall reaction, isovelocity, and small temperature difference. It is shown that a nearly frozen ignition regime also exists in the supersonic case. Ignition is not only governed by initial temperature as shown by Liñán and Crespo but also by shear parameters and detailed chemistry. It is important that the shear dissipation evolves heat, enhances the transport process, and accelerates the ignition very strongly. Increase of Mach number leads to the increase of ignition distance at low Mach numbers and to the decrease of ignition distance at high Mach numbers. Furthermore, it is shown that ignition is accelerated by an increase of branching reaction rates and is delayed by an increase of termination rate.

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